The Impact of Reduced Computational Complexity of Multiuser Detectors on the Processing Gain in a Wireless DS-CDMA Multiuser System

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Abstract - *In this paper, a new scheme for reducing the computational complexity of multiuser receivers is presented. It utilizes the transformation matrix (TM) algorithm to improve the performance of multiuser receivers by effectively reducing the bit error rate (BER). In addition, a deterministic formalization of the processing gain (PG) for a multiuser DS-CDMA system is presented. The proposed formalization of the PG demonstrates that how the reduced BER could be used to achieve reasonable values of PG by which unwanted signals or interference can be suppressed relative to the desired signal at the receiving end. The proposed algorithms not only are shown to substantially improve the performance of the multiuser detectors by means of reduced BER but also have a much lower multi-access interference. The performance measure adopted in this paper is the achievable bit rate for a fixed probability of error (10^{-7}) and consistent values of the PG.*

Keywords: A Computational complexity, Processing Gain, Multi-access Interference, Transformation Matrix.

1 Introduction

Multiuser direct-sequence code division multiple access (DS-CDMA) detection is an important technology in wireless systems for improving both data rate and the network capacity. Verdu [1] proposed and analyzed the optimum maximum likelihood (ML) sequence detector which, unfortunately, is too complex for practical implementation, since its complexity grows exponentially as a function of the number of users. Multiuser detectors suffer from their relatively higher computational complexity that prevents CDMA systems to adapt this technology for signal detection. In addition, one of the main characteristics that can severely degrade the performance of multiuser receivers is the inconsistent values of PG. However, if we could lower the complexity of multiuser detectors and produce better values of PG, most of the CDMA systems would likely take advantage of this technique in terms of increased system capacity and a better data rate.

In this paper, a novel approach for reducing the computational complexity of multiuser receivers is proposed that utilizes the TM algorithm to improve the performance of multiuser detectors. By using the proposed algorithm, the computational complexity of multiuser detectors can be reduced by several orders of magnitude. This is done by realizing that much of the processing performed is unnecessary. Since most of the decisions are correct, we can reduce the number of computations by using the transformation matrices only on those coordinates that are most likely to lead to an incorrect decision. By doing this, we can greatly reduce the processing that was required to make a decision about the correct region or the coordinate.

With the emergence of multiple access techniques, there has been an increase in the interest in performing simultaneous estimation and detection over all users [2]. Multiple access interference (MAI) can be prevented by selecting mutually orthogonal signature waveforms for all the active users. However, it is not possible to ensure perfect orthogonality among received signature waveforms, and thus MAI arises. In order to effectively eliminate MAI, this paper proposes a deterministic formalization of the PG for a multiuser DS-CDMA system. The proposed formalization demonstrates that how the reduced BER could be used to achieve reasonable values of PG by which unwanted signals or interference can be suppressed relative to the desired signal at the receiving end.

The rest of this paper is organized as follows: Section 2 describes the related work. Section 3 presents the original ML and the proposed TM algorithm. The deterministic formalization of the PG for a DS-CDMA system is presented in sections 3.3. The simulation results of PG and BER performance are provided in section 4. Finally, section 5 concludes the paper.

2 Related work

It has been shown in [5] that DS-CDMA is not fundamentally MAI limited and can be near-far resistant. In order to mitigate the problem of MAI, Verdu [5] proposed and analyzed the optimum multiuser detector for asynchronous Gaussian multiple access channels. The optimum ML receiver searches all the possible demodulated bits in order to find the decision region that maximizes the correlation metric given by [1]. The practical application of this mechanism is limited by the complexity of the receiver [6]. This optimal detector outperforms the conventional detector, but unfortunately its complexity grows exponentially with a complexity of $O(2)^K$, where K is the number of active users.

Much research has been done to reduce this receiver's computational complexity. Recently, Ottosson and Agrell [4] proposed a ML receiver that uses the neighboring decent (ND) algorithm. They implemented an iterative approach using the ND algorithm to locate the region where the actual observations belong. The linearity of the iterative approach increases noise components at the receiving end.

Several tree-search detection receivers have been proposed in the literature [12, 13], in order to reduce the complexity of the original ML detection scheme proposed by Verdu. Specifically, [7] investigated a tree-search detection algorithm, where a recursive, additive metric was developed in order to reduce the search complexity. Reduced tree-search algorithms, such as the well known M-algorithms and T-algorithms were used by [8] in order to reduce the complexity incurred by the optimum multiuser detectors. In addition, an optimal MMSE receiver requires the inversion of a large matrix. This computation takes a relatively long time and makes the detection process slow and expensive [10, 11]. Xie, Rushforth, Short and Moon [8] proposed an approximate MLSE solution known as the pre-survivor processing (PSP) type algorithm, which combined a tree search algorithm for data detection with the aid of the recursive least square (RLS) adaptive algorithm used for channel amplitude and phase estimation. MLSE receivers give optimum performance but at a cost of increased receiver computational complexity.

3 The computational complexity of multiuser receivers

We consider a synchronous DS-CDMA system as a linear time invariant (LTI) channel. In an LTI channel, the probability of variations in the interference parameters, such as the timing of all users is extremely low. The individual transformation points can be used to determine the average computational complexity. The system may consist of K users. User k can transmit a signal at any given time with the power of W_k. With the binary phase shift keying (BPSK) modulation technique, the transmitted bits belong to either +1 or -1, i.e., $b_k \in \{\pm 1\}$.

3.1 The ML algorithm

When a receiver wants to detect the signal from user-1, it first demodulates the received signal to obtain the base-band signal. The base-band signal is multiplied with user-1's unique signature waveform, $C_1(t)$. The resulting signal, $r_1(t)$, is applied to the input of the matched filter. The outputs of the matched filter can be represented by $y_k(m)$ and $b_k(m)$, respectively where m is the sampling interval. The outputs of the matched filter for the first two users at the m^{th} sampling interval can be expressed as follows:

$$y_1(m) = \frac{1}{T}\left\{\int_{(m)T}^{(m+1)T} r_1(t) C_1(t) dt\right\} \tag{1}$$

$$y_2(m) = \frac{1}{T}\left\{\int_{2+(m)T}^{2+(m+1)T} r_2(t) C_2(t-\tau_2) dt\right\} \tag{2}$$

The received signal $r_1(t)$ and $r_2(t)$ can be expressed as follows:

$$r_1(t) = \left(E_{C_1}\right)^{0.5} \sum_{i=-M}^{M} b_1(i) C_1(t - iT_b) \tag{3}$$

$$r_2(t) = \left(E_{C_2}\right)^{0.5} \sum_{i=-M}^{M} b_2(i) C_2(t - iT_b - \tau_2) \tag{4}$$

where E_{C_1} and E_{C_2} represent the original bit energy of the received signals. Substitute (3) and (4) as an individual equation into (1) and (2), respectively, and we get

$$y_1(m) = \frac{1}{T}\left\{\int_{(m)T}^{(m+1)T} \left(E_{C_1}\right)^{0.5} \left\{\sum_{i=-M}^{M} \left\{b_1(i) C_1\left(t - iT_b\right)\right\}\right\} C_1(t) dt\right\} \tag{5}$$

$$y_2(m) = \frac{1}{T}\left\{\int_{(m)T}^{(m+1)T} \left(E_{C_2}\right)^{0.5} \left\{\sum_{i=-M}^{M} \left\{b_2(i) C_2\left(t - iT_b - \tau_2\right)\right\}\right\} C_2\left(t-\tau_2\right) dt\right\} \tag{6}$$

After performing integration over the given interval, we get the following results with both noise components and the cross correlation of signature waveforms.

$$y_1(m) = \left(E_{C_1}\right)^{0.5} b_1(m) + \left(E_{C_2}\right)^{0.5} b_2(m-1)\rho_1 + \left(E_{C_2}\right)^{0.5} b_2(m)\rho_0 + \left(E_{C_2}\right)^{0.5} b_2(m+1)\rho_{-1} + n_1(m) \tag{7}$$

$$y_2(m) = \left(E_{C_2}\right)^{0.5} b_2(m) + \left(E_{C_1}\right)^{0.5} b_1(m-1)\rho_1 + \left(E_{C_1}\right)^{0.5} b_1(m)\rho_0 + \left(E_{C_1}\right)^{0.5} b_1(m+1)\rho_{-1} + n_2(m) \tag{8}$$

where coefficients $b_1(m)$ and $b_2(m)$ represent MAI, $\rho_{-1/0/+1}$ are cross-correlations of signature waveforms, and $n_1(m)$ and $n_2(m)$ represent the minimum noise components. These symbols can now be decoded using a ML Viterbi decision algorithm. The number of operations performed in the Viterbi algorithm is proportional to the number of decision states, and the number of decision states is exponential with respect to the total number of users. The computational complexity of this algorithm can be approximated as: $O(2)^{k}$.

3.2 The transformation matrix algorithm

According to original Verdu's algorithm, the outputs of the matched filter $y_1(m)$, *and* $y_2(m)$ can be considered as a single output $y(m)$. In order to minimize the noise components and to maximize the received demodulated bits, we can transform the output of the matched filter, and this transformation can be expressed as follows: $y(m) = \mathrm{T}b + \eta$ where T represents the TM, $b_k \in \{\pm 1\}$ and η represents the noise components. In addition, if the vectors are regarded as points in K-dimensional space, then the vectors constitute a constellation diagram that has K total points. This constellation diagram can be mathematically expressed as: $X = \{\mathrm{T}b\}$ *where* $b \in \{-1,+1\}$ where X represents the collective computational complexity of a multiuser receiver. According to the detection rule [1], the constellation diagram can be partitioned into 2^K lines (where the total possible lines in the constellation diagram can be represented as $\int$) that can only intersect each other at the following points: $X = \{\mathrm{T}b\}_{b \in \{-1,1\}^K} \backslash \int$.

Fig. 1 shows the constellation diagram that consists of three different vectors with the original vector 'X' that represents the collective complexity of the receiver. Q, R, and S represent vectors or transformation points within the coverage area of a cellular network as shown in Fig. 1. These three vectors can be viewed as three users that may exist anywhere within the coverage area of a cellular network. In addition, Q¬, R¬, and S¬ represent the computational complexity of each individual transformation point. The computational complexity of each individual transformation point is represented by $X\neg$ of the transformation point which is equal to the collective complexity of Q¬, R¬, and S¬. We consider the original vector with respect to each transmitted symbol or bit.

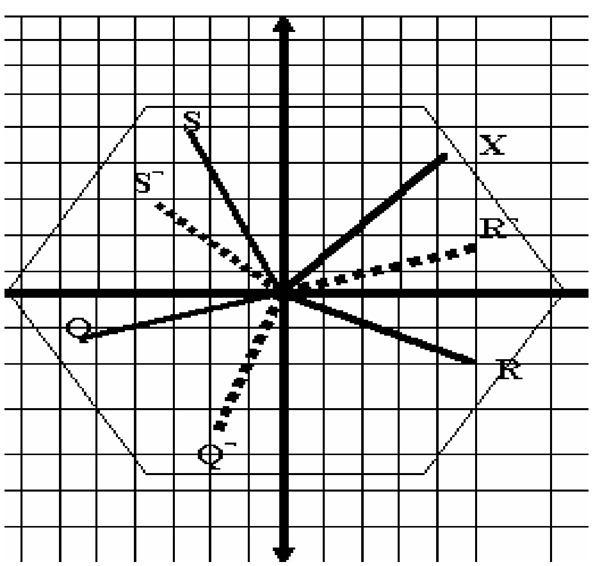

Figure 1. A constellation diagram consisting of three different parameters

$$X\neg Q = Xi\neg = \left(XQ_i + XR_j + XS_K\right) i\neg$$

$$X\neg R = Xj\neg = \left(XQ_i + XR_j + XS_K\right) j\neg$$

$$X\neg S = Xk\neg = \left(XQ_i + XR_j + XS_K\right) k\neg$$

The following equation can be derived from the above system:

$$\begin{pmatrix} X\neg Q \\ X\neg R \\ X\neg S \end{pmatrix} = \begin{pmatrix} ii\neg & ji\neg & ki\neg \\ ij\neg & jj\neg & kj\neg \\ ik\neg & jk\neg & kk\neg \end{pmatrix} \begin{pmatrix} XQ \\ XR \\ XS \end{pmatrix} \quad (9)$$

Equation (9) represents the following: *QRS* with the unit vectors $i, j,$ and k, and $X\neg Q$, $X\neg R$, and $X\neg S$ with the inverse of the unit vectors $i\neg, j\neg,$ and $k\neg$. The second matrix on the right hand side of (9) represents *b*, where as the first matrix on the right hand side of (9) represents the actual TM. Therefore, the TM from the global reference points to a particular local reference point can now be derived from (9):

$$\begin{pmatrix} X\neg Q \\ X\neg R \\ X\neg S \end{pmatrix} = T_{L/G} \begin{pmatrix} XQ \\ XR \\ XS \end{pmatrix} \quad (10)$$

Equation (10) can also be written as:

$$T_{L/G} = \begin{pmatrix} ii\neg & ji\neg & ki\neg \\ ij\neg & jj\neg & kj\neg \\ ik\neg & jk\neg & kk\neg \end{pmatrix} \quad (11)$$

We need to compute the locations of the actual transformation points described in (10) and (11). Let the unit vectors for the local reference point be:

$$\begin{aligned} i\neg &= \left[t_{11} i, t_{12} j, t_{13} k \right] \\ j\neg &= \left[t_{21} i, t_{22} j, t_{23} k \right] \\ k\neg &= \left[t_{31} i, t_{32} j, t_{33} k \right] \end{aligned} \quad (12)$$

since, $i\neg (i + j + k) = i\neg$, where $(i + j + k) = 1$. The same is true for the rest of the unit vectors. Therefore, (12) can be rewritten as:

$$\begin{aligned} i\neg &= \left[t_{11}, t_{12}, t_{13} \right] \\ J\neg &= \left[t_{21}, t_{22}, t_{23} \right] \\ k\neg &= \left[t_{31}, t_{32}, t_{33} \right] \end{aligned} \quad (13)$$

By substituting the values of $i\neg, j\neg,$ and $k\neg$ from (13) into (11), we obtain,

$$T_{L/G} = \begin{pmatrix} i(t_{11}i+t_{12}j+t_{13}k) & j(t_{11}i+t_{12}j+t_{13}k) & k(t_{11}i+t_{12}j+t_{13}k) \\ i(t_{21}i+t_{22}j+t_{23}k) & j(t_{21}i+t_{22}j+t_{23}k) & k(t_{21}i+t_{22}j+t_{23}k) \\ i(t_{31}i+t_{32}j+t_{33}k) & j(t_{31}i+t_{32}j+t_{33}k) & k(t_{31}i+t_{32}j+t_{33}k) \end{pmatrix} = \begin{pmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{pmatrix} \quad (14)$$

Substituting $\boldsymbol{T}_{L/G}$ from (14) into (10), yields

$$\begin{pmatrix} X & \neg & Q \\ X & \neg & R \\ X & \neg & S \end{pmatrix} = \begin{pmatrix} t_{1\,1} & t_{1\,2} & t_{1\,3} \\ t_{2\,1} & t_{2\,2} & t_{2\,3} \\ t_{3\,1} & t_{3\,2} & t_{3\,3} \end{pmatrix} \begin{pmatrix} X\ Q \\ X\ R \\ X\ S \end{pmatrix} \quad (15)$$

Equation (15) corresponds to the following standard equation used for computing the computational complexity at the receiving end: $X = \{\mathrm{T}b\}\ where\ b \in \{-1, +1\}^K$. Using (15), a simple matrix addition of the received demodulated bits can be used to approximate the number of most correlated transformation points. The entire procedure for computing the number of demodulated bits that need to be searched out by the decision algorithm can be used to approximate the number of most correlated signals. This is because we need to check whether or not the transformation points are closest to either (+1, +1) or (-1, -1). If the transformation points do not exist in the region of either (+1, +1) or (-1, -1), then it is just a matter of checking whether the transformation points are closest to (+1, -1) or to (-1, +1). In other words, the second matrix on the right hand side of (15) requires a comprehensive search of at most 5^K demodulated bits. This implies that the total number of demodulated bits that need to be searched out by the decision algorithm can not exceed by 5^K-4^K. This implies that both quantities T and b can be computed together and the generation of all the values of the demodulated received bits b can be done through the sum of the actual TM that approximately takes O $(5/4)^k$ operations.

3.3 Proposed deterministic formulization of the PG

It is observed that the PG has no effect on wideband thermal noise. In addition, a spread system requires the same transmitter power as an un-spread system on the additive white Gaussian noise channel (AWGN) when MAI is absent. However, we consider an AWGN channel where the MAI can affect the BER performance. For a DS-CDMA system, the PG can be viewed as ratio between the signal power and the interference power at the receiver. In a DS-CDMA based cell communication system, the interference is caused due to the cross-correlation between the spreading code as well as the number of users. Based on the above discussion, we can give the following mathematical hypothesis:

$$PG = S_P / I_P \quad (16)$$

where S_P and I_P represents signal and interference power, respectively.

As we know that the signal to noise ration (SNR) can be defined as a ration between signal power and noise power. By taking this into account, this relationship can be expressed as:

$$N_P \propto BER \quad (17)$$

where N_P represent the noise power. Based on (17), we can say that the effect caused on the SNR due to the values of N_P is the same as the effect caused by the BER values. Therefore, (17) can also be written as:

$$N_P \triangleq BER \quad (18)$$

where "$\triangleq$" represents the estimated value of a quantity for some large number '*n*'. By using (17) and (18), we can rewrite (16) as:

$$PG = (SNR)(BER)/I_P \quad (19)$$

The BER in (19) satisfies the characteristics of (17) and (18) for a high values of BER. Consider the following equation that can be used to determine the BER in an AWGN with the BPSK modulation technique.

$$\mathrm{BER} = Q\left[1/\sqrt{1/SNR}\right]$$

Since the attenuation factor and the white noise are uncorrelated, the SNR can be directly placed in the above equation as follows:

$$\mathrm{BER} = Q\left[1/10(SNR) + \sigma^2\right]^{-1/2} \quad (20)$$

where $Q(x)$ is the Gaussian Q function [3]. Recall (20) for an AWGN channel where the transmitted bits are modulated using the BPSK modulation technique, equation (19) can also be written as:

$$PG = (SNR)Q\left[1/10(SNR) + \sigma^2\right]^{-1/2} / I_P \quad (21)$$

where Q represents an error function. The second term of (21), σ^2, represents MAI that caused due to the cross corelation between the spreading code and the number of users and can vary due to the variations in the network load for an AWGN channel. As mentioned earlier, the interference power I_P represents MAI which is caused to the variation in network load. For simplicity, this can also be written as:

$$PG = (SNR)Q\left[\sqrt{10SNR}/\sqrt{1+10\sigma^2 SNR}\right]/I_P \quad (22)$$

Counter proof for the proposed model: We provide a proof for analyzing the correctness of (22) by considering the same set of derivations for computing the PG on an AWGN channel in the absence of MAI. We expect that the absence of MAI leads us to an equivalent mathematical equation like (22). In a spread-spectrum system, PG can be defined as:

$$PG = SNR_{PR} / SNR_{UP} \quad (23)$$

where subscripts PR and UP stand for *processed* and *unprocessed* signals respectively.

According to our initial assumption, the cellular network is modeled as a LTI system that permits us to clearly distinguish the unprocessed input signal to the processed output signal This, therefore, allows us to ignore the possibility of noise at the input signal. This leads us to the following mathematical hypothesis:

$$PG = 1/N_P = SNR/S_P \tag{24}$$

We use the same hypothesis that we presented to derive (17) and (18) in order to derive (25).

$$S_P \triangleq (1/BER) \Rightarrow PG = SNR(BER) \tag{25}$$

Recall our previous derivations of BER for an AWGN channel, (25) can also be written as:

$$PG = (SNR)\,Q\left[\sqrt{10SNR}/\sqrt{1+10\sigma^2 SNR}\right] \tag{26}$$

Equation (26) gives the PG when MAI is not caused due to the variation in the network load. However, the presence of MAI which is caused due to the cross corelation can not be ignored. By comparing (26) with (22), we can observe that the values of PG in (26) is higher than the values we can get from (22) because of the absence of MAI in (26). According to our initial assumption, if k_{th} signal changes during the transition, the output of the correlator is given as:

$$\Re_k = \mathrm{A}e^{-j\theta} + s_k N + \eta_k \tag{27}$$

In (27), the first, second, and third term represent the MAI component, the desired signal component, and the noise component, respectively. Our model is not affected by a phase shift and frequency shift. Therefore, this simplifies (27) as:

$$\Re_k = s_k N + \eta_k + A \tag{28}$$

where N is usefully interpreted as the PG. The second term in (28) is a zero mean Gaussian random variable with variance. The third term of (28) is a MAI component that can be defined as:

$$A = \sum_{k=2}^{K} A_k U_k \cos(\phi_k) \tag{29}$$

where A_k represents the envelop of a complex Gaussian process with unit variance and U_k represents a non-faded amplitude of the k_{th} signal. In (29), ϕ_k is a uniform random variable that represents the phase difference of the k_{th} user. To approximate the PG and MAI in the received signal, equations (28) and (29) can be used as follows:

$$\Re_k = s_k\left[(SNR)\,Q\left[\frac{\sqrt{10SNR}}{\sqrt{1+10\sigma^2 SNR}}\right]\right] + \sum_{k=2}^{K} A_k U_k + \eta_k \tag{30}$$

The first two terms of (30) can be used to approximate the PG and the MAI for a user k. The second term gives an average variance of the MAI over all possible operating conditions that can be used to compute the required SNR for a desirable BER performance. This can also be used in the reverse order by providing the required SNR for a known BER performance and computing the values of achievable PG. We use (30) in conducting the simulation result and performing the experimental verification by giving the non-folded amplitude of the k_{th} signal and computing the corresponding MAI as a Gaussian random variable with zero mean and conditional variance that represents by σ^2. Similarly, first terms of (30) can be used to approximate the values of achievable PG with respect to the required SNR for a desirable BER performance.

4 Performance analysis of the proposed algorithm

This section presents an analysis of the proposed deterministic formulization. In addition, we analyze the BER performance of the proposed algorithm and compare it to the ND and the ML algorithms. The system is modeled in MATLAB and the results are presented for both lightly and heavily loaded networks.

4.1 Analysis of the processing gain (PG)

The proposed deterministic formalization of the PG can yield significant improvements in term of receiver's PG (typically 4 to 10 dB). We computed this dB-Gain by comparing the differences of SNR values for both the proposed algorithm and the ND algorithm with the ML algorithm for a desirable BER performance. One way to decrease the MAI is to increase the power of the transmitted signals. Since we assume that we have a perfect power, it will not be practical to transmit signals with more power than we assume. Even though, we have made some strict assumptions about the perfect power control, the proposed algorithm still gets approximately 4 to 7 dB performance improvement over the ND algorithm. In (30), we present an approximation of PG and the MAI for the characteristic function involving both the *Q* function and the SNR. Since the *Q* function is widely available in many scientific software tools such as Matlab, components of (30) can be programmed for direct evaluation as shown in Fig. 2 and 3. It should be noted that the MAI has a Gaussian-like shape that decays exponentially with respect to the non-faded amplitude of the k_{th} signal as shown in Fig. 2. It can be seen that the convergence of MAI is exponential due to the divergence in the PG which is extremely a desirable property for a cellular network. Thus the results of Fig. 2 shows that the second term of (30) are extremely well behaved in a sense that they are smooth, strictly non negative, and decay exponentially due to the higher values of PG. In harmony

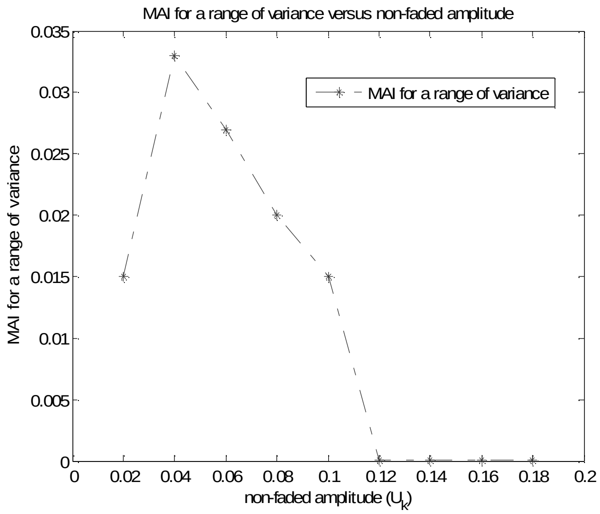

Figure 2. MAI for a range of σ^2 with K=5, and SNR=14 dB, N is computed using the first term of (30)

with our expectations, as the number of users, K, increased, the PG of the system degraded as shown in Fig. 3. This degradation in the PG is caused due to the decrease in SNR which consequently degrades the rate at which MAI diverges with respect to the PG. Fig. 4 demonstrates the PG improvement for a lightly-loaded network where K consists of 2 to 50. For a lightly-loaded network, an average of approximately 3.03 dB improvement is achieved by the proposed algorithm over the ND algorithm. The 3 dB of PG improvement in terms of SNR has significance, since this value can change the BER performance by approximately two orders of magnitudes. In other words, every time when we double the power level, we will add 3 dB to the current value of the power level. This corresponds to a 50 percent gain which is quite significant in cell communication. Fig. 5 shows the PG improvement for a heavily-loaded network with 50 to 100 users. The average PG improvement for this scenario is approximately 5.7 dB. If we compare the average PG of a heavily-loaded network with a lightly-loaded network, the

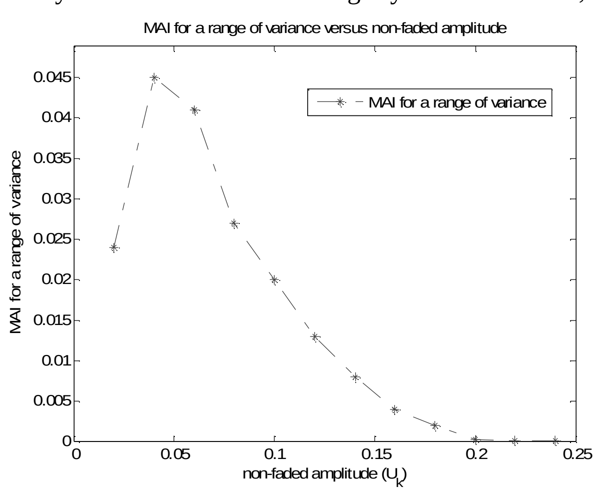

Figure 3. MAI for a range of σ^2 with K=10, and SNR=12 dB, N is computed using the first term of (30)

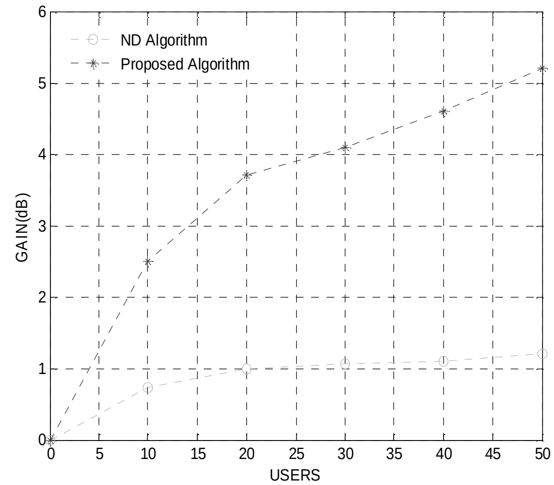

Figure 4. dB-Gain for a lightly-loaded network

proposed mathematical expression gets approximately 2.67 more PG improvement.

4.2 Performance Analysis of BER

Simulation results show that the proposed algorithm performs better than the ML and the ND algorithms for all values of BER. Fig. 6 and 7 show a plot of BER versus SNR curves. These curves were plotted using (21) in an AWGN channel for a small range of users. The simulation results for a lightly-loaded network demonstrate that an optimal BER performance can be achieved for a reasonable range of SNR. It should be noted that the BER performance of the proposed algorithm is always better than the ML and the ND algorithms as shown in Fig. 6. For the first few values of SNR, the ND algorithm almost approaches the ML algorithm where as the proposed algorithm still maintains a reasonable performance difference. It can be seen in Fig. 6 that the proposed algorithm achieves less than 10^{-2} BER for SNR = 8 dB which is quite closed to the required reasonable BER

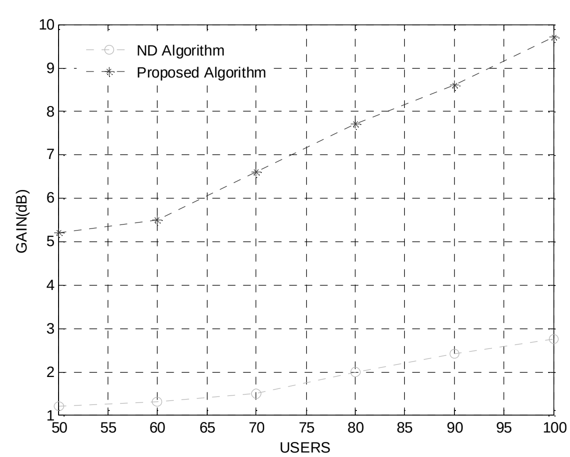

Figure 5. dB-Gain for a heavily-loaded network

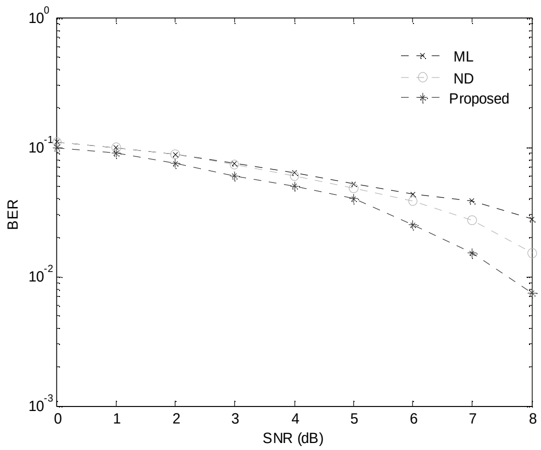

Figure 6. BER versus SNR (0<dB<9) curves

performance. For small values of SNR, the BER for these three algorithms is almost equal, but as we increase the value of SNR, typically more than 10 dB, we can observe the difference in the BER performance. The former result demonstrates a slight improvement over the BER performance shown in Fig. 6 for all SNR values above 9 dB. As the value of SNR increases, the BER performance of the proposed algorithm over the ND and the ML algorithms becomes more and more substantial because the probability of having more divergent values of SNR increases. It can also be noticed in Fig. 7 that the proposed algorithm achieves less that 10^{-3} BER for SNR = 10 dB which is more than what we desire for a voice communication system. Furthermore, the proposed algorithm achieves 10^{-7} BER performance for SNR = 14 dB as shown in Fig. 7.

5 Conclusions

In this paper, a new approach for reducing the computational complexity of multiuser receivers is proposed. Furthermore, we proposed a deterministic formulization of the PG for DS-CDMA systems. The main advantage of the proposed scheme is that it can be used for quantifying the MAI for a desirable BER performance. In order to show the consistency and the correctness of the proposed deterministic formulization, we presented simulation results for computing dB-gain with different ranges of users. The simulation results of PG demonstrated that the unwanted signals or MAI can be reduced relative to the desired signal at the receiving end. In addition, the simulation results for BER suggested that the proposed algorithm achieves better BER performance for all values of SNR than the other multiuser detection algorithms.

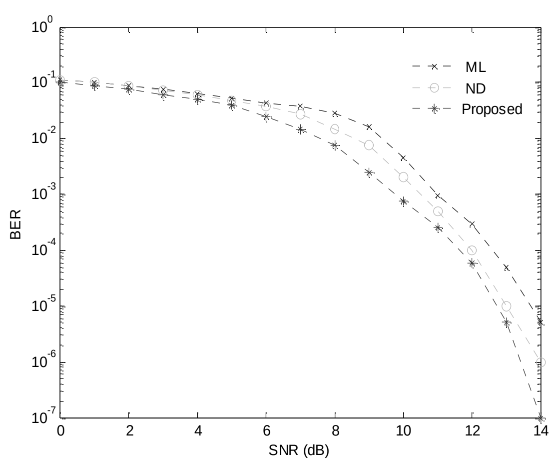

Figure 7. BER versus SNR (0<dB<14) curves

6 References

[1] S. Verdu, *Multiuser Detection*. Cambridge University Press, 1988.

[2] P. Tan and K. Lars, “Multiuser Detection in CDMA: A Comparison of Relaxations, Exact, and Heuristic Search Methods,” July 09, 2003.

[3] J. Proakis, “Digital Communications,” McGraw Hill International Editions, 3rd ed., 1995.

[4] T. Ottosson and E. Agrell, “ML optimal CDMA Multiuser Receiver,” Electronics Letters, Vol. 31, Issue-18, pp. 1544-1555, August 1995.

[5] S. Verdu, “Minimum probability of Error for Asynchronous Gaussian Multiple access Channels,” IEEE Transaction on Information Theory, Vol. IT-32, Issue-1, pp. 85–96, January 1986.

[6] G. Woodward and B. Vucetic, “Adaptive Detection for DS-CDMA,” Proceedings of the IEEE, Vol. 86, No. 7, pp. 1413 -1434, July 1998.

[7] L. Rasmussen, T. Lim, and M. Aulin, “Breadth-first Maximum Likelihood Detection in Multiuser CDMA,” IEEE Transactions on Communications, Vol. 45, pp. 1176-1178, October 1997.

[8] L. Wei, L. Rasmussen, and R. Wyrwas, “Near Optimum Tree-Search detection Schemes for Bit-Synchronous Multiuser CDMA Systems over Gaussian and Two-Path Rayleigh-Fading Channels,” IEEE Transactions on Communications, Vol. 45, pp. 691-700, June 1997.